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Application of Potential Method to survey analysis

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Abstract. This paper examines an alternative method for analyzing a collection of Likert items in the multi-criteria decision framework. Likert items are compared in pairs and organized in a set of weighted digraphs aggregated according to the Potential Method rules. In combination with Factor Analysis this approach gives respondents preferences on the scale which approximates a measurable value function. As an application of the proposed methodology, we examine a potential set of incentives and explore a degree to which they would be accepted by the industry. We use the Potential Method to elicit firms preferences for given incentives and seek to explain the difference in these preferences by the firm/market factors. Data is collected through a survey of 190 Croatian enterprises performed in 2002.

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1. Introduction

Public opinion polls are becoming extremely popular thanks to the existing communication possibilities and computer network. One of the most common format for measuring respondents' perceptions is the multiple-choice question and Likert type format (Likert [28]). This format captures the most valuable choice for the respondent, and no information regarding the relationship between the other possible choices are available. To overcome this lack of information several methods for item response formatting are proposed. The two most promising methods seem to be the *World Values Survey* [26], which uses the ranking scale, and the *Analytic Hierarchy Process* (AHP) (Saaty [36]), which uses positive reciprocal matrix for capturing the respondents' opinion. Both methods are time-consuming in the sense that raising the number of alternatives requires more time to complete the survey. This is especially true for the AHP response item format, which includes all pairwise comparisons among the attributes/options in the survey question.

The *Feeling Thermometer Scale* (FTS) used by the group of researchers from Stanford and Michigan [3] help respondents to express their perception on the "temperature" scale (0-100). "This method helps respondents clarify their preference

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precisely; however, consistency among responses to the alternatives is not always satisfactory" (Sato [37]).

If the response item format is of Likert type, a natural idea is to transform the survey data to the pairwise comparison format and use AHP or some other technique for analysis. The AHP matrix format seems to be inadequate for that purpose because the transformation from a 1-5 scale to the 1-3-5-7-9 scale used by Saaty, which measures the intensity of the preference is not obvious at all. Such transformation may loose the information and, moreover, it may include some extra noise in the data. Dittrich [18] used the *Adjacent Categories model*, Böckenholt and Dillon [9] and Agresti [1] postulate a power relationship between the response category and the probability of preferring one item to another.

Our contribution to the literature is that we preserve the traditional Likert item format and use the *Potential Method* (PM) in the survey analysis. The Potential Method shares the same underlying ideas as AHP and has no requirements in terms of data completeness which is also the case in our survey. While transforming Likert-type data to the preference graph required by PM, we use the majority preference (Čaklović [14]) as the intensity of preference, instead of the probability of choosing one item over another in the offered pairwise comparison among two items, which was the case in the work of Dittrich [18]. Roughly speaking, PM aggregates the intensity of the preference between two items, not the plurality of the item. In the situation when there is no missing data it may be proved that PM and additive aggregation[‡] give the equivalent rankings (Čaklović [17]). The details about PM are explained in the Appendix.

Another contribution is the way we combine the Potential Method with standard multivariate techniques for survey data analysis such as factor analysis. Namely, apart from questions which relate to the choice of alternatives, we have another set of questions which seeks to develop profiles of respondents using factor analysis. Factor analysis scores enter as importance weights into the potential method applied to the first set of questions.

The paper is organized in the following way. Sections 2 and 3 describe the focus of our application and the structure of the survey data. Section 4 explains in detail the procedure of data analysis. The findings are discussed in Section 5 and the conclusion remarks are given in Section 6. Finally, the Potential Method and aggregation procedures are explained in the Appendix.

2. The problem

The problem we choose for application of our new methodology is choosing optimal instruments for fostering industrial R&D in company. We explore tax incentives, tax incentives for collaborative projects with academia, direct subsidies for collaborative projects with academia, and government financing for employment of PhD degree holders in the industry.

Tax incentives have been utilized in many countries. Since R&D activity increases the propensity of firms to collaborate with public research institutions (Fontana,

[‡]Likert scale is a special case.

Geuna and Matt [20]), tax incentives can have an effect on industry-science collaboration via an increase in firm's R&D intensity. Although economists have been skeptical about the effect of fiscal incentives on private R&D, the outcome seems to be positive. There is considerable evidence that the R&D tax credit caused a significant response in US corporate R&D behavior (see [22, 25, 5, 29]). McCutchen [30] found out that the tax credit caused an increase in R&D expenditures in the pharmaceutical industry, and has contributed to an increase in competitive R&D spending among the firms. Bloom, Griffith and Reenen [7] estimated an econometric model of R&D investment on tax changes and R&D spending in nine OECD countries over a 19-year period and found evidence that tax incentives are effective in increasing R&D intensity. Paff [32] showed that alternate incremental credits (AIC) have effects on the tax price of research and the R&D investment of firms.

Another type of incentives is funding through public R&D funds. Public R&D programs have generally been designed to support commercial R&D projects with large expected social benefits but with inadequate expected returns to private investors (Klette, Moen and Griliches [27]). They are considered to be an effective public policy instrument when knowledge spillovers exist, as in the case of collaboration with universities in 2006 (Feldman and Kelley [19]). According to the systems approach to innovation public funding given for collaborative programs has potential to facilitate knowledge diffusion and interactive learning among economic actors, such as firms, universities, suppliers, and research institutes (Clausen [10]). Regarding the effect of subsidies, despite some doubts positive results were found. For example, Bérubé and Mohnen [6] found out that firms using both R&D grants and R&D tax-credits are more innovative. Czarnitzki and Licht [13] and Czarnitzki and Fier [12] find that firms that received R&D subsidies spent more on innovation and R&D, and that subsidies influence firms patenting activities in a positive way (Czarnitzki and Licht [13]). Clausen [10] empirically shows that "research" subsidies stimulate R&D spending within firms while "development" subsidies substitute such spending.

In this paper we also examine industry preference toward some measures for improving employment of new PhDs in industry. Many countries implement mobility programs that allow companies to contract researchers from public research centers and universities (Almeida, Dokko and Rosenkopf [2]). These scientists bring knowledge previously developed in the public R&D system that enables the firm to access new knowledge and improve the exploitation of existing knowledge. Herrera, Munoz and Nieto [24] show that scientific knowledge which public researchers provide has positive influence on both inputs and outputs of the firms' innovation process. A special facet of mobility concerns new PhDs and their employment in the industry. Since most valuable knowledge often has a large tacit component which is embedded in people, an effective way for organizations to acquire desired knowledge is to hire individuals who possess it (Argote and Ingram [4]). Although the industry can accomplish knowledge transfer by hiring PhDs, companies may be reluctant to hire them. Wright, Clarysse, Lockett and Knockaert [40] show that large companies are more likely to hire PhDs, while this is more difficult for Small and Medium Businesses (in the further text SMEs) due to constrained resources and a lack of awareness on the part of researchers of available opportunities.

3. Data

The data for this study were collected in the spring of 2002. The survey work was preceded by exploratory research, during which in-depth interviews were conducted with R&D directors from ten firms. Exploratory research was crucial in design of the survey instrument.

Two hundred and thirty (230) firms were chosen for the survey. Those firms were registered as performing some technology-related activities, and also as having invested in R&D in the time period between 1997 and 1999[§]. The latter ensured that only active firms were included. Out of 230 firms that were targeted, 190 responded. This represents the response rate of 82.6%. The survey instrument was a questionnaire, and respondents were R&D directors of selected companies who were surveyed over the telephone.

A method for measuring respondents perceptions is the traditional five-level balanced Likert response format which captures the degree of their attitude for a given item (Tables 1 and 2). For some questions multiple answers are acceptable (Table 3).

	Question
I1.	To what extent do you collaborate with academics? (not at all) 1 2 3 4 5 (very intensely)
I2.	How would you rate the quality of that collaboration? (very unsatisfactory) 1 2 3 4 5 (very satisfactory)
I3.	How would you rate a commercial effect of this collaboration? (very unsatisfactory) 1 2 3 4 5 (very satisfactory)
I4.	In your company, innovations are very important. (totally disagree) 1 2 3 4 5 (totally agree)
I5.	In your company, new technologies are very important. (totally disagree) 1 2 3 4 5 (totally agree)
I6.	Your company has access to most advanced technologies. (totally disagree) 1 2 3 4 5 (totally agree)

Table 1: *Input variables*

3.1. Input variables

Questions that relate to firms' characteristics are presented in Table 1. They are referred to as *input variables*, because they form input into the model. These variables came out of the exploratory research, where they were identified as important factors on which firms differed and which affected their attitude toward possible incentives.

[§]Those 230 firms represented the total population of firms in Croatia which satisfied both conditions.

3.2. Output variables

We have two sets of *output variables*. The first set of *output variables* measures the attitude toward financial incentives while the second set relates to mobility incentives (Table 2). Each of the statements O1a, O1b, O1c, O2a, and O2b is rated on the scale from 1 (totally disagree) to 5 (totally agree). Specifically O1a and O1b refer to tax

Question	Answers
O1. To what extent do you agree that the following incentives can improve collaboration between the industry and academics? Scale used: 1 (totally disagree)...5 (totally agree)	O1a. Tax cuts for companies for investing in their own R&D. O1b. Tax cuts for companies for investing in joint R&D projects with academic institutions. O1c. Direct involvement of governmental agencies through partial financing of joint collaborative projects between the industry and academics.
O2. To what extent do you agree that the following incentives for improvement of mobility of PhDs could improve collaboration between industry and academics? Scale used: 1 (totally disagree)...5 (totally agree)	O2a. Government financing of PhD program for selected young industry employees who after several years spent at the academic institution and obtained degree return to their company. O2b. Two year co-financing for the first time employment of new PhDs in industry, so that government pays 50% of their full salary in the first year and 30% in the second year.

Table 2: *Output variables: incentives*

incentives. We differentiate between tax cuts for one's own R&D and tax cuts which are targeted specifically to aid collaborative programs with academia. Variable O1c refers to the direct R&D subsidies for collaborative projects. Variables O2a and O2b measure the attitude toward incentives for improving employment of PhDs. The difference between them is that O2a refers to situation when industry sends selected employees to a university to obtain PhD before returning to the company, while O2b considers a situation when a new PhD (who did not necessarily have any prior relation to industry) is to be employed in the industry.

Question	Answers
O3. Who should initiate collaboration between industry and academics? (Multiple answers are acceptable)	O3a. industry O3b. academics O3c. governmental agencies

Table 3: *Output variables: initiator*

To shed additional light on collaboration, in this paper we consider not only the desirable incentives for industry-science collaboration, but also the party which should initiate it. Table 3 describes *output variable* O3. The information about the initiator of the collaboration can provide an additional insight into the optimal policies.

4. Methods and analysis

4.1. Factor analysis

The respondents in the survey are firms whose "profiles" are constructed from Table 5, using 6 previously described input variables. More precisely, we would like to link characteristics of the firms with their preferences for policy measures, but we need to assign weights to those input variables. Before we do that, we will examine possible correlations among groups of variables, the reason being that some of these variables could measure the same thing. For that purpose we use factor analysis.

Factor analysis is a data reduction technique used to investigate whether a group of variables has common underlying dimensions and can be considered to measure a common factor. Although the analysis can be used to summarize a larger number of variables into a smaller set of constructs, ultimately the analysis is not a hypothesis testing technique so it does not tell us what those constructs are (Hanley, Meigs, Williams, Haffner, D'Agostino [23]). In turn, the validity of naming the constructs is contingent upon researcher judgment and should be interpreted with some caution (Thompson-Larry [39]). For factor analysis we used principal components analysis followed by Varimax rotation. As typical, factors with eigenvalues less than one are dropped from further analysis just like variables with factor loadings of less than 0.6 as these are not considered statistically significant for interpretation purposes.

Questions	Innovations **Factor F1**	Collaboration **Factor F2**	weight
I1. To what extent do you collaborate with academics?	0.225492	**.900.667657**	0.138089
I2. How would you rate the quality of this collaboration?	0.125369	**.900.828172**	0.171288
I3. How would you rate the commercial effect of collaboration?	0.045920	**.900.834602**	0.172618
I4. In your company, new technologies are very important.	**.900.826936**	0.074072	0.177905
I5. Your company has access to most advanced technologies	**.900.875878**	0.103873	0.188434
I6. Your investors are willing to support your innovation efforts	**.900.704978**	0.178838	0.151667
Eigenvalues	2.02	1.88	
Variance explained	33.6%	31.3%	
Cumulative variance explained	33.6%	64.9%	

Table 4: *Input variables and factor loadings*

Factor analysis on variables I1,..., I6 finds two factors (Table 4). The first factor explains the largest amount of variance and addresses to the importance the firm places on innovations and new technologies, and on the firm's innovative efforts compared with competitors. The second factor refers to firm's extent of collaboration and the satisfaction with collaboration and its commercial effect.

Before we continue, we need to explain the use of weights in the analysis. Usually in MCDM weights are used so that they denote importance of certain variables/attributes. Therefore it is customary in the decision process that larger weight is placed upon attributes of larger importance. In this paper we modify this approach by choosing weights that reflect the extent to which the variables/criteria explain differences among firms with respect to their opinion about policy measures.

The reason for our modification is that we want firms that have experience and propensity for collaboration to have larger influence on the policy outcome. This is because specialist knowledge providers such as universities and other research institutions are more likely to be engaged by firms with more open approaches to innovation, those with high levels of absorptive capacity, those with greater networking capabilities, as well as by those with deeper commitments to innovation (Tether-Tajar [38]). Therefore we feel it is more reasonable to give weaker voice to the firms that have weak absorptive capacity, networking skills and low commitment to innovation, and to give stronger voice to the firms on the other end of the spectrum. In order to accomplish that, instead of using weights to represent importance, we use them to reflect the extent to which a firm is innovation and technology oriented, and to which it collaborates with research institutions. In this way the firms that engage in collaboration and are committed to innovation will have greater power in indicating good directions with respect to the proposed policy measures.

We form weights for variables I1,. . . , I6 in the following way: we consider the factor the variable belongs to, and form the weight for that variable by multiplying the factor's eigenvalue by the factor loading for that variable within that factor. For example, for variable I1 which belongs to the factor *Collaboration* the corresponding weight is $1.88 \cdot 0.67$. In this way we assign weights according to the amount of variance explained by the factor (eigenvalue) and the correlation of a variable with the factor (factor loading). The weights thus reflect the extent to which variables explain differences among firms related to industry-science collaboration, innovation and technologies, customers and investors. Table 4 presents the weights.

FIRM	I1	I2	I3	I4	I5	I6
Firm 1	4	4	4	1	2	5
Firm 2	3	4	4	4	4	3
Firm 3	1			2	2	1
Firm 4	4	5	3	5	4	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Firm 189	5	3	3	5	5	3
Firm 190	2	5	3	3	3	3

Table 5: *Input for the 1^{st} run. Decision table.*

4.2. Application of the Potential Method to the problem

After the weights are determined, next step is the ranking of the firms using the Table 5 and the weights of the variables calculated by factor analysis. There is missing data in the table which could otherwise present a problem, however Potential Method has ability to deal with this issue easily. (Čaklović, Šego [17, 15]).

The Table 5 can be interpreted as a decision table with the variables I1 to I6 as states of the nature, and firms as actions. To each variable we will assign a preference flow according to formula (3). Aggregated flow is computed according to formula (2), taking into account variable weights obtained from factor analysis. Overall priorities of the firms are calculated according to formula (11) from [16, p. 4] and (4).

Once the ranking of the firms is accomplished, the final step is the ranking of the *output variables* using the obtained rankings of the firms. The ranking procedure and hierarchical structure of the problem is presented in Figures 1 and 2.

$$\begin{tikzcd}[column sep=huge]
\textbf{Factors} \arrow[r, "\text{Question}"] \arrow[d, swap, "\text{Factor}"', "\text{analysis (Table 4)}"] & \textbf{Output vars} \text{ (Tables 2 and 3)} \\
\textbf{Input vars} \arrow[r, swap, "\text{PM (1}^{\text{st}}\text{ run) (Table 5)}"] & \textbf{Firms} \arrow[u, "\text{PM (2}^{\text{nd}}\text{ run)}", "\text{(Table 7)}"']
\end{tikzcd}$$

Figure 1: *Steps in the ranking procedure of output variables*

Before aggregation, each individual flow may be re-normalized multiplying its components by the non-negative number called *flow-norm* (FN), see the discussion in Appendix. The ranking of actions (firms) remains the same regardless of which value of FN we choose. Different values of FN would produce the same ranking whith slight changes in the numerical values of the ranks. For the purpose of this analysis we used the value FN=2 for each criterion (input variable). The ranking of *output*

	Firm 1	Firm 2	···	Firm 189	Firm 190
O3A	1	1	···		1
O3B		1	···		1
O3C	1	1	···	1	

Table 6: *Input for 2^{nd} run. Variable O3.*

	Firm 1	Firm 2	···	Firm 189	Firm 190
O1A	5	4	···	5	4
O1B	5	4	···	5	4
O1C	5	5	···	5	4
O2A	5	5	···	5	5
O2B	5	3	···	4	5

Table 7: *Input for 2^{nd} run. Variables O1 and O2.*

variables is done in the 2^{nd} run of PM in the same way as above using the data from Tables 6 and 7. In those tables the firms are playing a role of states and output variables are actions.

To recapitulate, we used PM in two steps in solving the problem of consensus on incentives. These steps are presented in Figure 1.

For better understanding we visualy represent the above procedure in Figure 2. The first level of the hierarchy contains factors F1 and F2. Their relative weights are 2.01 and 1.88 respectively. Factor loadings within each factor we interpret as relative weights of the variables and use them to define weights of the variables I1–I6 as explained in the previous section (Figure 2, second level). We then use them to rank companies (Figure 2, third level). Companies' rankings are used as weights to rank the output variables at the bottom of the the hierarchy (Figure 2, fourth level).

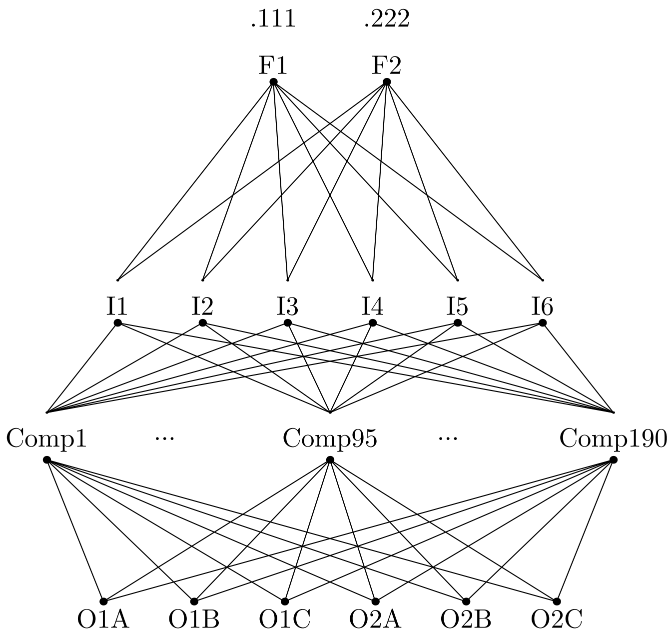

Figure 2: *Decision hierarchy*

5. Findings

The resulting ranks are presented in column *global ranking* of Table 8. By global ranking we mean rankings when both questions O1 and O2 are considered together. To check the robustness of results (i.e. stability of relative order of alternatives O1A, O1B, O1C, O2A and O2B) questions O1 and O2 were considered separately (i.e. we performed local ranking). From Table 8 we observe that the ranking within questions remains the same in local as in global ranking. This indicates robustness of the method.

nodes	rank	nodes	rank	options
O1A	0.314	O1A	0.518	• tax breaks for own R&D
O1B	0.280	O1B	0.352	• tax breaks for joint R&D
O1C	0.216	O1C	0.130	• gov. subsidies for collaborative projects
O2B	0.112	O2B	0.800	• co-financing of employment for new PhDs
O2A	0.078	O2A	0.200	• gov. financing for PhD from industry
global ranking		local ranking		

Table 8: *Final rankings – O1 & O2*

When all types of instrument are considered, question O1 is ranked above O2. This means that companies rank financial instruments higher than instruments related to the employment of PhDs.

When we bear in mind that question O1 is about financial resources, it is not so surprising that it carries the most weight with the companies. Extant literature shows that lack of financing for R&D and innovation is one of the major causes of problems in innovation development (Mohnen, Palm, Loeff and Tiwary [31]). Regarding particular incentives (Table 8), data shows that tax breaks are ranked at the top of the list. More precisely, tax breaks to firms for performing R&D and tax breaks for collaborative projects are ranked higher than the third option which is direct involvement of governmental agencies. If we look at this order closely, we can observe that the preference for an incentive increases with the degree of autonomy that the beneficiary enjoys.

The most preferred instrument allows the most autonomy to the firm. Under that scheme the firm is just to perform R&D with no strings attached. Naturally, companies prefer that instrument since it allows the most freedom. According to extant literature, tax incentives have positive effect on R&D spending in companies, and higher R&D intensity is expected to lead to increase in collaboration (Fontana, Geuna and Matt [20]). In further support for this policy instrument, some studies show that academics also believe that strengthening of R&D in companies would prompt industry to seek collaboration with academia (Radas, Vehovec [33]).

Tax breaks for specific purpose, i.e. for collaborative projects, are ranked lower. We can explain this by observing that before this scheme can be applied, the company needs to go through the effort of finding the right partner and the right project. The required effort makes this particular incentive more costly for companies. This instrument will be less likely to help in brokering the first contact between potential partners than to help those companies that already have well defined partnerships.

The least favorite option among financial incentives is government subsidy for collaborative projects. This last option leaves even less freedom to the firm, and involves the third party (government). The companies may not want the government to interfere because this introduces additional rigidities in the process which may be complicated enough¶.

¶As a check, we compared the results of the PM method with the results of the ranking of alternatives O1A, O1B and O1C. Namely, in the questionnaire we asked an additional question where respondents were requested to rank order the alternatives O1A, O1B and O1C by giving 3 points to the most preferred option and 1 to the least preferred. The ranking resulted in O1A being ranked

Regarding the instruments related to PhD employment, data shows that it has the least degree of relevance compared to other instruments. This may be due to the fact that this instrument does not bring immediate direct financial benefits to the firm. The benefit of these instruments is for the firm to be able to afford employing a PhD, which down the road is supposed to yield financial benefits through products, services and processes founded on the new knowledge.

In particular, the incentive O2B is ranked higher than O2A. This can be explained by the stress imposed on the organization which is created by selecting an employee and sending it away from company for an extended period of time (this may include reorganizing work, hiring another temporary employee, etc). That may also indicate that the tacit knowledge acquired by an individual is too valuable for the firm to loose, even if it is just for a limited time. Also, by choosing to employ an outside PhD, the firm gets to choose from the fresh pool of knowledge and new networking contacts. By using the incentive O2B, the firm shares the cost of job training for the new PhD during the years when the financial results stemming for integration of this new knowledge can be expected to be small.

nodes	**rank**	**options**
O3A	0.583	• industry
O3B	0.271	• academics
O3C	0.146	• gov. agencies

Table 9: *Final rankings – O3*

An important issue for structuring of incentives is who should initiate the collaboration (Table 9). Data shows that companies prefer that industry should be given the initiating role, while academics come second on the ranking and government is placed last. Again companies show large preference for autonomy. Due to the applied nature of collaborative projects, it really falls upon the industry partner to commercialize the outcome of the collaboration. Consequently, the company also bears the risk of failure. Judging themselves better acquainted with the market needs, companies naturally consider themselves also better equipped to choose the topic and initiate the collaboration.

On the technical note, the variables O3A, O3B, O3C are ranked calculating the Condorcet flow and applying the PM to it. The reason for that is the ordered scale which is used to measure the answers for those questions. See section 7.4 for details.

Firms' preferences can depend on their size. This is why we repeated the above analysis for SMEs and large firms separately. We did not find any differences in ranking of instruments (the relative rankings do not change).

6. Conclusion

In this paper we combined Potential Method with factor analysis to overcome some issues present in the current analysis of survey data. The obtained scale is a measurable value function, i.e. the distance between the items can be determined, which

first, O1B second and O1C third. This confirms the results obtained from the PM method.

opens a myriad of possibilities for further analysis. Our paper fits in the stream of research which deals with the survey data analysis and applications of the recently developed techniques in the decision making framework.

As an application of proposed methodology, in this paper we examine companies' preference for several policy instruments related to improvement of industrial R&D. We examine three possible instruments: tax incentives for R&D, government subsidies for joint R&D projects with academia, and mobility incentives for new PhDs. We use Potential Method approach which shares the same idea of pairwise comparisons as AHP, but is better suited for dealing with Likert items, especially in the presence of missing data. Missing data appears when respondents fail to give answers to one or more questions, resulting in deletion of those respondents and consequently decrease in the sample size. This can be prevented by using Potential Method as we show in the paper.

We also show how Potential Method can be combined with multivariate techniques such as factor analysis. The key step in the proposed approach is to use factor analysis to obtain the hierarchical decomposition of the respondents and their weights. Those weights are used in aggregation of output variables over the respondents. Incentive output variables scale is obtained by aggregation of the preference flows while the initiator output variables sale is obtained by aggregation of the ordinal flows because of the ordinal nature of the questionnaire. This idea seems not to be used yet in the survey analysis framework.

The proposed combination can draw out respondent preferences for offered alternatives taking in account respondents characteristics together with their choices. This produces deeper insights into the analyzed problem.

7. Appendix. Potential Method

To avoid the self-plagiarism we strongly suggest the reader to read the article *Measure of Inconsistency for the Potential Method* [16]. To introduce the notation, we shall only repeat here the definition of the preference graph and preference flow.

7.1. Preference flow

A `preference graph` is a *directed graph* (or *digraph*) $\mathcal{G} = (V, \mathcal{A})$ where V is the set of nodes and $\mathcal{A}$ is the set of *directed edges* (or *arc*) of $\mathcal{G}$. We suppose that $\mathcal{G}$ has no parallel arcs nor loops. Let us denote $m = \operatorname{Card} \mathcal{A}, n = \operatorname{Card} V$.

We say that node $a \in V$ is `more preferred` than node $b \in V$, in notation $a \succcurlyeq b$, if there is an arc $\alpha \in \mathcal{A}$ such that a is the terminal (ingoing) node and b is the initial (outgoing) node of the arc α. We also say that α *leaves* b and *enters* a. The set of arcs represents a subset (relation) of the Cartesian product $V \times V$. In our notation[‖],

$$a \succcurlyeq b \iff (a, b) \in \mathcal{A}.$$

[‖]The reason for taking $\alpha = (b, a)$ and NOT $\alpha = (a, b)$ is because of the notation $(a \leftarrow b)$ for exchange between b and a in the construction of measurable value function [21, p. 82].

The `incidence matrix` $A = (a_{\alpha,v})$ of the graph $\mathcal{G}$ is $m \times n$ matrix, where

$$a_{\alpha,v} = \begin{cases} -1, \text{ if } \alpha \text{ leaves node } v \\ \ \ 1, \text{ if } \alpha \text{ enters node } v \\ \ \ 0, \text{ otherwise.} \end{cases}$$

We shall write a_{ij} where i is the index of i-th arc and j is the index of j-th node. The vector space $\mathbb{R}^m$ is called `arcs space` and the vector space $\mathbb{R}^n$ is called `vertex space`. It is an elemetary fact that the incidence matrix A (and matrix in general) generates an orthogonal decomposition

$$N(A^\tau) \oplus R(A) = \mathbb{R}^m \tag{1}$$

where $R(A)$ is the column space of the matrix A and $N(A^\tau)$ is the null-space of the matrix A^τ. The space $N(A^\tau) \subset \mathbb{R}^m$ is called `cycle space` because it is generated by all cycles of the graph.

In decision making, to each arc (preference) we often associate an intensity of that preference, on some scale, which motivates the following definition:

Definition 1. *A* `preference flow` *is a non-negative real function $\mathcal{F}$ defined on the set of arcs. For the arc $\alpha = (a, b)$, $\mathcal{F}_\alpha = 0$ means that the decision maker is* indifferent *to the pair $\{a, b\}$. In that case orientation of the arc is arbitrary.*

The preference flow $\mathcal{F}$ can be considered as an element of the arcs-space and identified by the column vector of length m. From the computational point of view it is useful to put $\mathcal{F}_{-\alpha} = -\mathcal{F}_\alpha$ if $\alpha = (u, v)$ is an arcs.

An example of a preference graph in a voting procedure was considered by Condorcet. He defined the `social preference` flow as

$$\mathcal{F}_C(u, v) := N(u, v) - N(v, u)$$

where $N(u, v)$ denote the number of voters choosing u over v.

7.2. Aggregation of flows

In group decision we have a finite number of decision makers, members of the group, and each of them has his own preference graph over the set of alternatives. In practice, each member of the group can have his own hierarchy with one restriction, that the nodes of the bottom level (alternatives) are the same for all members. The procedure of aggregation of individual flows into the group flow we call `consensus`, often used notion in the literature is *social preference*. Aggregation procedure remains the same if instead of a group we have two or more criteria or goals.

The procedure of making a `consensus graph` $(V, \mathcal{A})$ and `consensus flow` is the following. Each criterion $C_i, i \in \{1, \ldots, k\}$ generates its own preference graph $(V, \mathcal{A}_i)$ and its own preference flow $\mathcal{F}_i$. Let w_i denote the weight of i-th criterion.

First, for the given pair $\alpha = (u, v)$ of alternatives we calculate

$$F_\alpha := \sum_{\substack{i=1 \\ \pm\alpha\in\mathcal{A}_i}}^{k} w_i \mathcal{F}_i(\alpha) \tag{2}$$

where the item $w_i\mathcal{F}_i(\alpha)$ is taken into account if and only if $\alpha \in \mathcal{A}_i$ or $-\alpha \in \mathcal{A}_i$. If this sum is non-negative, then we include α in the set $\mathcal{A}$ of arcs of the consensus graph, and we put $\mathcal{F}(\alpha) := F_\alpha$. If it is negative, we define $-\alpha = (v, u)$ as an arc in $\mathcal{A}$ and $\mathcal{F}(-\alpha) := -F_\alpha$. The flow $\mathcal{F}$ becomes non-negative. If F_α is not defined then u and v are not adjacent in the consensus graph.

In MCDA the trade off between scales, measured by different criteria, should be done. In PM it is done by normalizing the max-norm of the flow $\mathcal{F}$ to the prescribed value called *flow-norm* (in notation FN). This concept allows decision maker to increase or decrease a difference between ranks without changing the original input. We suggest to decision maker to try several values of FN before making consensus if necessary. FN is a unification parameter for different measure units accompanied with each criterion.

7.2.1. Decision table

For a decision table the consensus flow, defined on the graph with actions $a_1, \dots a_m$ as nodes and with states $\theta_1, \dots \theta_n$ as criteria, according to the formula (2), is defined by

		States of nature			
		θ_1	θ_2	$\cdots$	θ_n
Actions	a_1	v_{11}	v_{12}	$\cdots$	v_{1n}
	a_2	v_{21}	v_{21}	$\cdots$	v_{21}
	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$
	a_m	v_{m1}	v_{m2}	$\cdots$	v_{mn}

Table 10: *Decision table*

$$\mathcal{F}_{kj} = \sum_i P(\theta_i)(v_{ki} - v_{ji}), \quad k, j = 1, \dots, m \text{ and } k \neq j. \tag{3}$$

Here $\mathcal{F}_{kj}$ denotes the flow component on (a_k, a_j).

7.2.2. Hierarchical decision

In a hierarchical decision structure each node, except nodes on the last level, is a parent node for its children (leaves) from some other level. A parent node may be considered as a criterion for evaluation of its children. The only restriction is that children of the parent should be in the same level. The parents of a node can be from different levels. Restriction, made by the conservation law, is that the sum of the weights of nodes in some level set should be the sum of the weights of their parents.

PM calculates the weights of nodes in some level in the following way. First, the weight of the goal is set to be 1. For a particular level which is not yet ranked, the aggregation of flows is made over the set of all parents, potential X is calculated after that the weights w are obtained using the formula

$$w = k \cdot \frac{a^X}{\|a^X\|_1} \tag{4}$$

where $\|\cdot\|_1$ represents l_1-norm and k is the sum of weights of the parents (usualy k=1). The process is repeated until the bottom level of the hierarchy is ranked.

The exponential function $X \mapsto a^X$ is defined by the components and $a > 1$ is a positive constant. Currently, we use the value $a = 2$ but user may precise some other value. If a preference graph is not connected the above procedure should be done for each connected component.

7.3. Incomplete data structure

For *Potential Method* 'missing pairwise comparison' means that the nodes are not adjacent in the preference graph. For instance, if some cells in decision Table 10 are empty the sum in the formula (3) will avoid the corresponding indices in the flow construction.

7.3.1. An example of incomplete table

As an example of incomlete data structure let us consider the Table 5. For the sake of simplicity we shall calculate the preference flow and the corresponding potential for the first four rows of the table. Individual flows for each input variable are given in the Table 11. They are calculated according to formula (3). Evidently, some individual flows are not complete but the composite flow $\mathcal{F}_c$ is complete. Relative weights of the variables are given in the second row. The composite flow $\mathcal{F}_c$ is a 'linear combination' of renormalized individual flows according to formula (2), while renormalization of the flow is done by fixing the maximal component of the flow to a given value (2 in this case). Finaly, we calculate potential X_c and the weights of each row using the formula (11) from [16, p. 4] and formula (4). The results are given in the Table 12 bellow:

	Individual flows						composite
weights	1/20	3/20	5/20	2/20	4/20	5/20	flow
Arcs	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$\mathcal{F}_c$
$1 \leftarrow 2$	1	0	0	-3	-2	2	-0.27
$1 \leftarrow 3$	3			-1	0	4	0.55
$1 \leftarrow 4$	0	-1	1	-4	-2	2	-0.15
$2 \leftarrow 3$	2			2	2	2	0.82
$2 \leftarrow 4$	-1	-1	1	-1	0	0	0.12
$3 \leftarrow 4$	-3			-3	-2	-2	-0.90

Table 11: *Individual flows for Table 5*

Node (row)	1	2	3	4
Potential X	0.03	0.30	-0.57	0.23
weight w	0.25	0.30	0.16	0.29

Table 12: *Results of the analysis of Table 5*

7.4. Aggregation of ordered lists

Aggregation of ordered lists is not an easy task. There are several excellent outranking methods which are developed for that purpose: PROMETHEE [8] ELECTRE type methods [35, 34] and others. For example, the answers of each company in the columns $O3A, O3B, O3C$ of the output Table 6 can be regarded as the ballots which is an ordered list of the possible options. The part of that table is given in Table 13. This means that the company C1 gave the ordered list of the options $\{O3A, O3B, O3C\}$ in order $(O3A = O3C, O3B)$, the company C2 in order $(O3A = O3B = O3B)$ and so on. A natural way to make the aggregation is to calculate the Condorcet flow [11] and apply the Potential Method to that flow. Another suggestion is to calculate relative frequency of each option in the list of all ballots and make the stochastic flow, cf. Čaklović [14]. All of them are equivalent in our case and give the same ranking.

Condorcet flow on the arc $A \leftarrow B$ is defined as the difference $n(A,B) - n(B,A)$ where $n(A,B)$ denotes the number of dominance of the option A over B in a set of ordered lists. In the Table 13 we have $n(O3A, O3B) = 2$, $n(O3B, O3A) = 2$, $n(O3A, O3C) = 2$, $n(O3C, O3A) = 3$, $n(O3B, O3C) = 2$ and $n(O3C, O3B) = 3$. The flow matrix of the Condorcet flow for options $\{O3A, O3B, O3C\}$ is calculated in the Table 14. The X-column of the table gives the values of the potential X calculated in the same way as above in the example of incomplete table. In our situation companies have different weights so that the intensity of the preference of $O3A$ over $O3B$, for example, is multiplied by the weight of the company which gives the priority. The ranking, obtained after application of PM to the weighted Condorcet flow, is given in Table 9.

	O3A	O3B	O3C	O1A	O1B	O1C
C1	1	0	1	5	5	5
C2	1	1	1	4	4	5
C3	1	1	0	5	4	4
C4	1	0	1	5	5	4
C5	1	1	1	5	5	5
C6	1	1	1	5	5	5
C7	0	1	1	3	4	4
C8	1	1	0	4	4	4
C9	0	1	1	4	3	3
C10	0	0	1	5	5	

Table 13: *Example – the first 10 companies*

	$O3A$	$O3B$	$O3C$	X (potential)
$O3A$	0	0	-1	$-1/3$
$O3B$	0	0	-1	$-1/3$
$O3C$	1	1	0	$2/3$

Table 14: *Flow matrix of the Condorcet flow for options* $\{O3A, O3B, O3C\}$ *in the above example*

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